Краткие сообщения ОИЯИ №4[67]-94 *JINR Rapid Communications No.4[67]-94*
УДК 539.17

QUASI-CLASSICAL DESCRIPTION OF ONE-NUCLEON TRANSFER REACTIONS WITH HEAVY IONS

S.I.Fedotov, V.K.Lukyanov

The heavy-ion one-nucleon transfer reactions are considered using the distorted waves obtained in the framework of the high energy approximation (HEA) method in the three-dimensional quasi-classics. The bound state nucleon wave function is presented in a form of the derivative of the Fermi function. The cross section is obtained in the analytic form, showing the main physical features of the reaction mechanism. The results of calculations are in good agreement with experimental data.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Квазиклассическое описание однонуклонных передач в реакциях с тяжелыми ионами

С.И.Федотов, В.К.Лукьянов

Рассматриваются однонуклонные передачи в реакциях с тяжелыми ионами с использованием искаженных волн, полученных в рамках метода высокоэнергетического приближения (ВЭП) в трехмерной квазиклассике. Волновая функция связанного состояния нуклона выбирается в виде производной от ферми-функции. Дифференциальные сечения получаются в аналитическом виде, что позволяет понять основные свойства механизма реакций. Результаты расчетов находятся в хорошем согласии с экспериментальными данными.

Работа выполнена в Лаборатории теоретической физики им.Н.Н.Боголюбова ОИЯИ.

1. Introduction

The traditional consideration of the high-energy transfer reactions with heavy ions is based on a partial wave representation of the *in*- and *out*-distorted waves. To this aim, at energies of several dozen MeV per nucleon, it is necessary to numerically calculate a lot of partial waves which introduce both the hard numerical problems and difficulties in searching for the physics of the reaction mechanism. To avoid these difficulties, we apply the HEA-method developed for calculations of the three-dimensional quasi-classical wave functions and for the corresponding matrix elements with these functions included [1,2]. The method can be applied under the conditions $kR \gg 1$, $E \gg V$ and $\theta > \theta_c \cong |V|/E$, where θ_c is the classical deflection angle. This latter is introduced to include distortion of the straigt-line trajectories of motion, the important point in investigating the heavy-ion collisions. On the whole, this gives us the possibility of avoiding complicated numerical calculations and obtaining, in the framework of the DWBA, analytical expressions for qualitative physical estimations and for a quantitative comparison with experimental data.

2. Differential Cross Section

For simplicity, we consider the reaction $a + A \to b + B$ with transfer of a spinless x-particle when the corresponding cross section and the amplitude in the zero-range approximation are as follows:

$$\frac{d\sigma}{d\Omega} = \frac{m_\alpha m_\beta}{(2\pi h^2)^2}\frac{k_\beta}{k_\alpha}\frac{2J_B + 1}{(2J_A + 1)(2J_a + 1)}\sum_l \frac{S_l}{2l+1}\left|\tilde{T}_l^{tr}\right|^2, \quad (2.1)$$

$$\tilde{T}_l^{tr} = -D_0 \int d\mathbf{r}\, \Psi_\beta^{(-)*}(\mathbf{r})\, \Psi_\alpha^{(+)}(\mathbf{r})\, \mathfrak{R}_l(r)\, Y_{l0}^*(\hat{r}), \quad (2.2)$$

where

$$D_0 = 8\pi\sqrt{(m_a h^2/2m_x m_b)^3 \varepsilon_{xb}}$$

depends on the structure of an incident particle, ε_{xb} is the separation energy and $\mathfrak{R}_l(r)$ is the radial wave function of the x-particle in the final nucleus B. The latter has the asymptotic behaviour $\exp(-k_l r)/r$ and goes to the constant as $r \to 0$. We have emphasized that the main effect in heavy ion reactions comes from the region near the interaction radius. This means that the behaviour of the function $\mathfrak{R}_l$ at $r \ll R$ is of no importance, and one can select it in the form

$$\mathfrak{R}_l(r) = \frac{\sqrt{6a_l}}{r}\frac{df_s(r, R, a_l)}{dr}, \quad (2.3)$$

where

$$f_s = \frac{\sinh\dfrac{R}{a_l}}{\cosh\dfrac{R}{a_l} + \cosh\dfrac{r}{a_l}} \cong \frac{1}{1 + \exp\dfrac{r-R}{a_l}} \quad (2.4)$$

is the symmetrized Fermi function having the asymptotics $\exp(-k_l r)/r$ and being a constant at $r = 0$; the function (2.3) is normalized to 1, and the «diffuseness» of the transition region is to be taken $a_l = 1/k_l$, where $k_l = \sqrt{2m_x \varepsilon_l / h^2}$ with ε_l, the separation energy.

Inserting (2.3) into (2.2), we get the amplitude of the typical form inherent in HEA. Moreover, here we can use the quasi-elastic approximation because the loss of energy in the reaction is comparatively small and $E_\alpha \cong E_\beta$. Thus, the QC-distorted waves are calculated as in the elastic channel. The product $\Psi_\beta^{(-)*}\,\Psi_\alpha^{(+)}$ has the following form [1,2]

$$\Psi_\beta^{(-)*}\,\Psi_\alpha^{(+)} = \exp(i\tilde{\Phi}), \tag{2.5}$$

where

$$\tilde{\Phi} = 2\bar{a}_0 + \tilde{\beta}\mu + n_1\mu^2 + c_1\mu^3 + \\ + n_2(1-\mu^2)\cos^2\bar{\varphi} + c_2\mu(1-\mu^2)\cos^2\bar{\varphi} \tag{2.6}$$

and $\tilde{\beta}$, c and n are expressed through r, parameters of the potentials, $\alpha = \sin(\theta/2)$, and $\alpha_c \cong \frac{1}{2E}[V(R_t) + V_C(R_t) + iW(R_t)]$, taken at the radius R_t of the external limited trajectory of motion. For example,

$$\tilde{\beta} = \tilde{q}r = q_{ef}r + 2k_\delta \alpha r; \qquad q_{ef} = 2k(\alpha - \alpha_c);$$

$$k_\delta = -\left[B^V + iB^W + B^C\left(3 - \frac{r^2}{R_C^2}\right)\right], \tag{2.7}$$

where

$$B^V = \frac{V_0}{hv}; \qquad B^W = \frac{W_0}{hv}; \qquad B^C = \frac{Z_1Z_2e^2}{R_C hv}.$$

The other functions in (2.6) are done in [1].

We can see that now the integrand (2.2) contains in the exponent a typical power dependence on the variables r and μ. Keeping in mind that $dr = -r^2 dr d\mu d\bar{\varphi}$, we first integrate in (2.2) over $d\mu$ by parts

$$I_l = \int_{-1}^{+1} d\mu \exp(i\tilde{\Phi})\, Y_{l0} \cong$$

$$\cong -i\left(\frac{\exp(i\tilde{\Phi})}{\partial\tilde{\Phi}/\partial\mu}\bigg|_{+1} - (-)^l \frac{\exp(i\tilde{\Phi})}{\partial\tilde{\Phi}/\partial\mu}\bigg|_{+1}\right) Y_{l0}(1), \tag{2.8}$$

neglecting the second term having the smallness $(kR)^{-2}$. The result is

$$I_l = -i\sqrt{\frac{2l+1}{4\pi}}\exp(2i\bar{a}_0 + in_1)\,[I^{(+)} - (-)^l I^{(-)}];$$

$$I^{(\pm)} = \frac{\exp[\pm i(\tilde{\beta} + c_1)]}{\Delta_{(\pm)} \mp \delta_{(\pm)}\cos^2\bar{\varphi}}; \tag{2.9}$$

$$\Delta_{(\pm)} = \tilde{\beta} + 3c_1 \pm 2n_1; \qquad \delta_{(\pm)} = 2(n_2 \pm c_2). \tag{2.10}$$

Then, the integration over $d\bar{\varphi}$ can be performed with the help of a table integral. Thus, we can write the amplitude (2.2) in the form of a one-dimensional integral [1,2]:

$$\tilde{T}_l^{tr} = -iD_0\sqrt{6\pi a_l(2l+1)}\,e^{i2\bar{a}_0}\int_0^\infty \frac{df_s}{dr}\{F^{(+)}(r) - (-)^l F^{(-)}(r)\}\,dr, \tag{2.11}$$

where

$$F^{(\pm)}(r) = \frac{\exp[\pm i\phi^{\pm}]}{L^{\pm}} = \exp[\pm i\Phi^{\pm}]; \tag{2.12}$$

$$\Phi^{\pm} = \phi^{\pm} - \ln L^{\pm}; \qquad \phi^{\pm} = f_1 r \pm f_2 r^2 + f_3 r^3;$$

$$L^{\pm} = \sqrt{(f_1 \mp f_4 r + f_5 r^2)(f_1 \pm f_6 r + f_7 r^2)} \tag{2.13}$$

with f, the functions of parameters of the potentials, α and α_c:

$$f_1 = 2k(\alpha - \alpha_c) - 2(B^V + iB^W + 3B^C)\alpha; \qquad f_2 = \left(\frac{B^V}{R_V} + i\frac{B^W}{R_W}\right)(1-\alpha^2);$$

$$f_3 = \frac{2B^C}{R_C^2}\left(1 - \frac{2}{3}\alpha^2\right)\alpha; \qquad f_4 = 2\left(\frac{B^V}{R_V} + i\frac{B^W}{R_W}\right)\alpha^2;$$

$$f_5 = \frac{2B^C}{R_C^2}\left(1 - 2\alpha^2\right)\alpha; \qquad f_6 = 2\left(\frac{B^V}{R_V} + i\frac{B^W}{R_W}\right)(1-2\alpha^2);$$

$$f_7 = \frac{2B^C}{R_C^2}(5 - 6\alpha^2)\,\alpha. \tag{2.14}$$

Integrals of the type (2.11) can be caclulated in the analytical form if one uses the second order poles on the complex r-plane of the derivative

df_s/dr in the region of the nuclear surface at $r_n^{\pm} = R \pm i\pi\,(2n+1)\,a_l$, where $n = = 0, 1, 2\ldots$ It is easy to show that the main contribution to (2.11) is coming from the two poles closest to the real axes of r. Then, the final expression for the differential cross section is as follows:

$$\frac{d\sigma}{d\Omega} = 6\pi a_l^3 S_l D_0^2 \frac{2J_B+1}{(2J_A+1)(2J_a+1)} e^{4B^W R_W} \left| \left[\frac{d}{dr}\exp(i\Phi^{(+)})\right]_{r_0^+} + \right.$$

$$\left. + (-)^l \left[\frac{d}{dr}\exp(-i\Phi^{(-)})\right]_{r_0^-} \right|^2. \qquad (2.15)$$

Thus, we can conclude that here we have the general exponential decrease at angles $\theta > \theta_c$, depending on the acting thickness a_l in the region of the surface of transition [3]. The magnitude of the cross section is determined by the slope of a «tail» of a bound state function in the final nucleus B.

3. Conclusion

Figure 1 shows calculations (solid line) and comparisons with experimental data (squares) from [4]. We can see that the differential cross section is decreasing with the angle of scattering as an exponential function with a slope determined by the thickness parameter a_l characterizing the corresponding form factor behaviour in the surface area of interaction. In Fig.1, the solid line corresponds to $a_l = 0.4$ fm. The spectroscopic factor was taken to equal 1. The other parameters are $V_0 = 50$ MeV, $W_0 = 19$ MeV, $r_{0v} = r_{0w} = r_{0c} = r_{0t} = 1.2$ fm.

In our calculations the absolute values of theoretical cross sections are presented. The absolute values of the cross sections and their form strongly depend on the imaginary part of the complex potential. In the case of a potential when W_0 is very small, the main part of differential cross section $\sim \exp(-2\pi a_l k(\Theta - \Theta_c))\cos^2(kR\Theta)$ for even l (solid line in Fig.2) and $\sim \exp(-2\pi a_l k(\Theta - \Theta_c))\sin^2(kR\Theta)$ for odd l (dashed line in Fig.2). We see that the cross section is decreasing with the angle of scattering as an

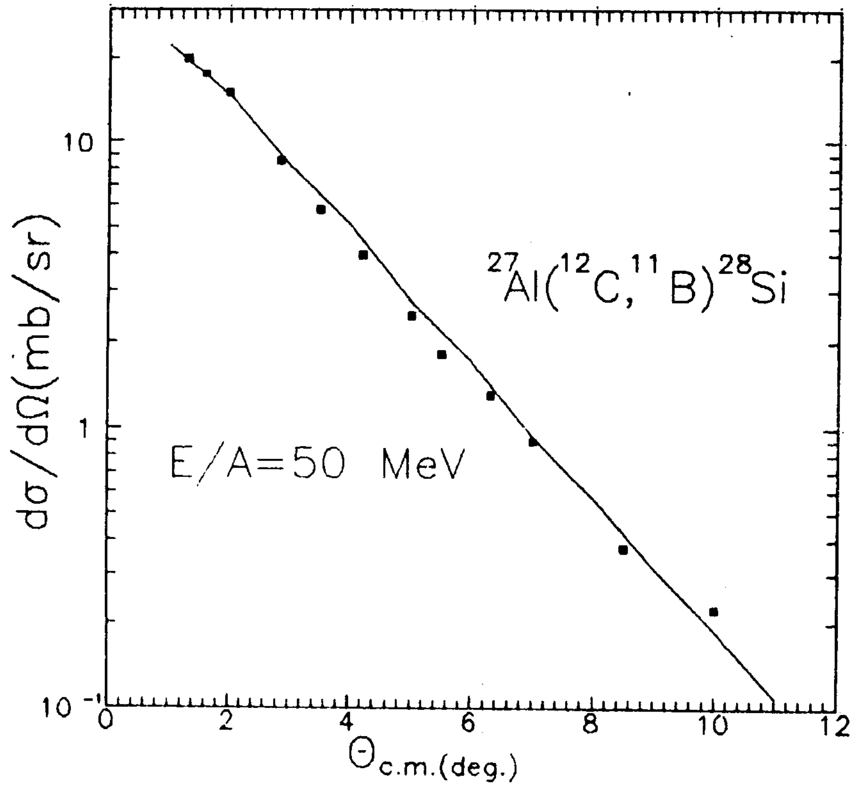

Fig. 1. The transfer reaction cross sections: $^{12}C + {}^{27}Al \rightarrow {}^{11}B + {}^{28}Si$; $E = 50$ MeV/n, exp. data (squares) from [4], solid lines — theory

exponential function and simultaneously oscillates. The oscillation with even l are out phase with those with odd l. The solid line with stars corresponds to $W_0 = 10$ MeV, when the oscillations start to appear.

We can summarize that investigations of heavy ion collisions in the quantum region of scattering angles $\Theta > \Theta_c$, outside the limited trajectories of motion, are very sensitive to the precise structure of a nuclear-nuclear interaction. For instance, the slope of the curves with Θ feels the «thickness» of the acting region in the corresponding channel. It may be used also for searching the «halo» distributions of nuclei in the radioactive beams which now become available. We hope that the HEA-method suggested can be successfully used in both the qualitative and quantitative analysis of direct reactions.

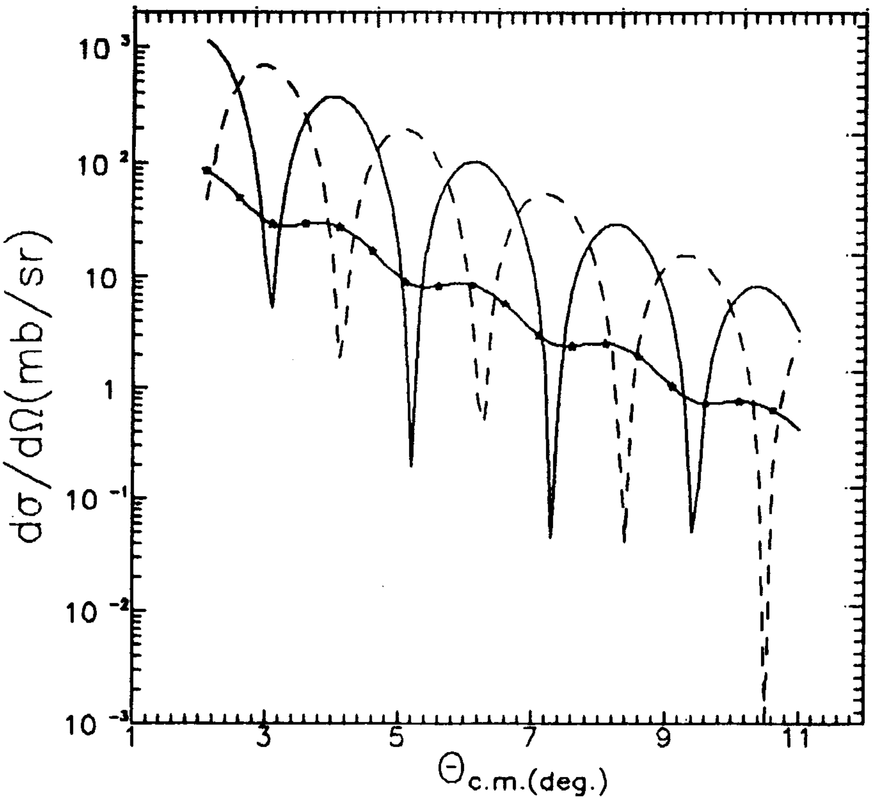

Fig.2. The solid line shows the cross section calculated for the same reaction by using W_0= 1 MeV, l = 2, the dashed line shows the cross section calculated by using W_0= 1 MeV, l = 1 and the solid line with stars shows the cross section calculated by using W_0= 10 MeV, l = 2

References

1. Lukyanov V.K. — JINR Preprint E4-93-314, Dubna, 1994.
2. Lukyanov V.K. — Proc. Int. School-Seminar on HI Phys., JINR E7-93-274, Dubna, 1993, v.2, p.129.
3. Lukyanov V.K., Gridnev K.A., Embulaev A.V. — Bull. of Russ. Acad. of Sc., Physics, 1994, v.58, p.19 (in Russian Izv. RAN, ser. fiz., 1994, v.58, p.23).
4. Winfield J.S. et al. — Phys. Rev., 1989, v.C39, p.1395.

Received on August 12, 1994.